

I. *A Dissertation concerning the Figure of the Earth.*
Part the Second. By the Reverend J. T. Desaguliers, LL. D. F. R. S.

SINCE my * Paper concerning the Figure of the Earth was read before the Royal Society, I met with a Dissertation of Monf. *Mairan* (in the Memoirs of the Royal Academy of *Paris*, for the Year 1720.) wherein the learned and ingenious Author has taken a great deal of Pains to reconcile the Observations made on Pendulums, (found to be shorter at the Æquator than at *Paris*, when they swing Seconds) with the oblong spheroidical Figure of the Earth, deduced from Monf. *Cassini's* Measures. And tho' upon a strict Examination of his Conjectures, and what he gives for Demonstrations, I do not find Reason to alter my Opinion concerning the oblate or flatted Spheroid, which Sir *Isaac Newton* has shewn to be the Figure of the Earth; yet since it might be thought by some, who have read Monf. *Mairan's* Treatise, and afterwards may read mine, that I have not consider'd all the Circumstances that He has done, and that I have not been exact enough in the Mathematical Part of my Dissertation, because I have drawn some Conclusions from supposing the Figure of the Earth spherical, when I should have suppos'd it an oblong Spheroid; I beg Leave to shew here, wherein I think Monf. *Mairan* is mistaken, and to give those additional Proofs of my Assertions, which I promis'd the Society when I gave in my last Paper.

* *Vid.* Philof. Transf. N^o. 386, 387.

First then I begin with the Conjectures.

Monf. *Mairan* fays, that it is as reasonable to fuppofe the Earth (if it was once fluid) to have been an oblong Spheroid at firft, as a Sphere; and that, in fuch a Cafe, the Centrifugal Force of the feveral Parts of the Earth, arifing from its Revolution about its Axis, which might convert a Sphere into an oblate Spheroid, wou'd only change an oblong Spheroid into one lefs oblong.

If the Earth was at firft a Fluid, (fuppofed homogeneous, and of any given Form,) and left to thofe Laws, which we find to obtain at prefent; it muft put on a fpherical Figure, for the fame Reafon that Drops of Mercury, of Water, and other Fluids, put on fuch a Figure. And to fuppofe any Change made in that Figure from the Prefsure of an external Fluid, filling up all Space, is contrary to what has been demonftrated by Sir *Ifaac Newton* in his *Principia Lib. 2. Prop. 19.* where he fhews, *That if any Portion of a Fluid be comprefs'd by the fame or any other homogeneous Fluid, that Portion will not have its Figure alter'd by that Prefsure.*

And indeed we fee, that in the Receiver of the Air-Pump, Lumps of Butter, coagulated Oil, or Honey, Drops of Quickfilver or Water, &c. have the fame Figure, whether the Prefsure of the Air acts upon them, or be taken off by exhaufting the Receiver.

That a fluid Substance, of any Figure, will by the Gravity of its Parts become fpherical, is plain by the following

DEMONSTRATION. *Fig. I.*

Let *ABCDE* be a Portion of an homogeneous Fluid, whose Parts tend towards one another, and whose Figure is

is not spherical. If in such a Fluid we suppose a Syphon as ACE (or which is the same thing, if all the Fluid shou'd be frozen, except the Canal ACE) whose Legs AC and CE are unequal, and meet at C, the Center of the Fluid, towards which there is the greatest Tendency; the Fluid will run out at A in the Leg AC, till it be come down as far as *g* in the Leg CE, supposing Cg equal to AC. But if the Leg AC be lengthened as far as *c*, then the Fluid will only come down as far as *e* in the Leg CE, and at the same time rise up to *a* in the Leg Ca, Ca being equal to Ce.

If such another Canal or Syphon be suppos'd at BCD, the Fluid in it will come down from D to *d*, and rise from B to *b*. And since such Syphons may be suppos'd all over the Fluid ABDE; that Fluid, by the mutual Tendency of its Parts towards one another, must be reduc'd to the spherical Figure *abde*. *Which was to be demonstrated.*

Now, without considering the Unreasonableness of the Supposition, let us imagine the Earth to have been an oblong Spheroid at first, and then to have a diurnal Revolution given to it, which shou'd by Degrees shorten its Axis, to bring it to what Messieurs *Cassini* and *Mairan* suppose it at present to be. If in such a Case the Earth be suppos'd fluid enough to change its Figure, by the Revolution about its Axis, why should it stop when the Æquatorial Diameter comes to want just one 96th Part of the Length of the Axis? since two Powers act upon it to shorten its Axis, *viz.* Gravity, and the Centrifugal Force; the first of which has already been shewn capable to reduce it to a Sphere, and the Centrifugal Force is acknowledged by Monf. *Mairan* to be (as Sir *Isaac Newton* has prov'd it) at the Æquator equal to $\frac{1}{230}$ Part of the Gravity there. Certainly the Alteration of Figure wou'd not have stopp'd, before the

Earth came to be a Sphere ; nay, and it must have risen at the Æquator ; and how much, I have already shewn in my former Paper.

Again, if we suppose the Earth of an heterogeneous Fluid, before the diurnal Revolution, the heaviest Parts wou'd go towards the Center, and the lighter towards the Surface ; and that Way the Terraqueous Globe wou'd also become a Sphere. Then if, when the Central Parts are fix'd, and the superficial *Strata* are still fluid, the Earth receives a diurnal Motion ; it will rise at the Æquatorial Parts, and that to a greater Height than what I have shewn in my former Paper, where I suppos'd the Earth of uniform Matter. And that something like this must be the Case, appears from what Sir *Isaac Newton* has said upon this Subject. For after having shewn, from supposing the Earth of uniform Matter, that the Centrifugal Force of all its Parts wou'd bring it to be $17\frac{1}{2}$ *English* Miles higher at the Æquator than at the Poles, and after having given a Table of the proportionable Decrease of the Length of the Degrees of a Meridian of the Earth, going from the Poles to the Æquator, in such a Figure of the Earth, with the Lengths that Pendulums must have to swing Seconds in several Latitudes ; from a Comparison of the Lengths of Pendulums (observ'd by different Persons to be shorter towards the Æquator, than in greater Latitudes (when they swing Seconds) he shews that the Earth must be $31\frac{1}{2}$ Miles higher at the Æquator than at the Poles ; and therefore that it must be denser towards the Central than the Superficial Parts to produce a flatted Spheroid, where the Æquatorial Diameter must exceed the Axis so much more ; that is, be longer something more than $\frac{1}{145}$ Part.

Lastly, let us suppose the Earth, at its first Creation, to have been made of Land and Water, the first as solid,
and

and the last as fluid as it is now, but of *Monf. Cassini's* Figure, and examine the Consequence. Since in that Figure the Axis is $\frac{1}{32}$ Part longer than the *Æquatorial* Diameter; the Gravity will be so much greater at the *Æquator* than at the Poles, that the Waters will all flow to the *Æquatorial*, and leave the Polar Regions; which will happen still more by the Centrifugal Force, which the Earth in its diurnal Motion will give to the Fluid; and therefore the Sea wou'd be $43\frac{2}{3}$ Miles (reckoning 5000 Feet to a Mile) higher at the *Æquator* than at the Poles, which must overflow all the *Torrid Zone*, and leave the Polar Regions dry.

I am very well aware, that it may be objected by such as have read *Monf. Mairan's* Dissertation, and have not read *Sir Isaac Newton's Principia*, or have not read that Book with due Attention — “ That I have not argued fairly in drawing Consequences from a greater Gravity at the *Æquator* than at the Poles, in an oblong Spheroid; because *Monf. Mairan* has shewn, that, in such a Figure of the Earth, the Gravity is greater at the Poles than at the *Æquator*; and that I shou'd have drawn my Consequences from these Principles.” To which I answer, that his Demonstrations about Gravity are built upon wrong Suppositions, as I shall shew by and by. Nevertheless, supposing that Gravity was greater towards the Poles than towards the *Æquator*, in the Proportion that he assigns, namely of the Ray of Curvature drawn into the Perpendicular to the Curve, terminated at the Axis; let us consider what will follow from his Principles.

* Let us then suppose the Earth at first in a fluid State; *AA* the Axis, *dÆ* the *Æquatorial* Diameter, *ob* a Ray of Curvature, *dn* another, *ac* and *dC* two Lines

of Tendency or Perpendiculars to the Curve, intercepted by the Axis at c and C ; and dC , AC , two Tubes or Canals of the Fluid, gravitating towards, and communicating at C . I say that, according to *Monf. Mairan's* Principles of Gravity, the Earth cannot preserve its oblong spheroidical Figure. For since the Gravity at a : Is to the Gravity at d :: As $dn \times dC$: to $ab \times ac$, it will follow (from the Nature of the Ellipse) that the Gravity at A : will be to the Gravity at d :: As AC^2 : to dC^2 : and therefore the Forces, with which the Columns of Fluid AC and dC tend towards C , will be as their Masses drawn into the Forces driving towards C , that is, as $AC \times AC^2$ to $dC \times dC^2$. Now by the Principles of Hydrostatics, it is evident that the Fluid, in the Canal AC , will cause the Fluid in the Canal dC to run out at d as long as $AC \times AC^2$ is greater than $dC \times dC^2$: And if the Canal Cd be continued quite to δ , the Surface of the Fluid in AC will sink to α , whilst the Surface of the Fluid in dC rises up to δ , in which Case as $\alpha C = C\delta$, the Point A will come to α , and the Point d to δ , and the Curve Ad being chang'd into $\alpha\delta$, the oblong Spheroid will be chang'd into a Sphere, the only Figure consistent with the *Æquilibrium* of the fluid Parts, according to *Monf. Mairan's* own Principles; because then you will have $AC^2 = dC^2$, and $AC \times AC^2 = dC \times dC^2$. If we make use of *Sir Isaac Newton's* Principles in this Reasoning, we shall also shew, that an oblong, spheroidical, fluid Earth will be chang'd into a Sphere; but not so fast as it does by *Monf. Mairan's* Laws; for, according to *Sir Isaac Newton*, the Gravity at A : Is to the Gravity at d :: As $\sqrt{Cd} : \sqrt{AC}$. Q. E. D.

NB. *Here we have suppos'd no diurnal Revolution, for as soon as that begins, the Centrifugal Force will raise the Æquatorial Parts, and change the Sphere into a flatted Spheroid, as has been before shewn, and is allow'd by Monf. Mairan.*

Now if we suppose the same Figure of the Earth, but the Land (at its first Creation) as firm as it is now ; it will in that Case follow from Monf. *Mairan's* Principles, that the Sea must rise and overflow all the Æquatorial Regions, tho' the Earth had no diurnal Revolution; and much more so, when the Centrifugal Force, arising from the diurnal Motion, helps to carry the Water the same Way.

DEMONSTRATION.

Let $P \alpha P \text{Æ}^*$ represent the Plane of a Meridian, PP the Axis of the Earth (suppos'd an oblong Spheroid) $\alpha \text{Æ}$ the Diameter of the Æquator, $d e a \alpha$ Part of the Surface of the Earth, αA and $e B$ two Perpendiculars to the Surface of the Earth (which are here two Rays of Curvature) $f c$ the Surface of the Sea, and $f d e g$, $b a \alpha c$ two Cylinders of Sea-Water of equal Bases and equal Heights.

Since Gravity acts on the two equal Columns of Water $b a c \alpha$, $f d e g$ in the reciprocal Ratio of the Ray of Curvature (at the respective Places of the Columns) drawn into that Part of it which Monf. *Mairan* calls the Line of Tendency, (that is, in the Ratio of $e B \times e Z$ to $\alpha A \times \alpha C$) the Weight of $f e$: will be to the Weight of $b \alpha$:: As $\alpha A \times \alpha C$: to $e B \times e Z$. Therefore if there be a Communication between the fluid Co-

* Fig. III.

lums fe and ba , there cannot be an *Æquilibrium*, till the Quantity of Matter in fe , becomes to the Quantity of Matter in ba , reciprocally as the Gravity at the Place a is to the Gravity at e ; and in that Case the Height ge will be reduc'd to ke , if $ke : ca :: eB \times eZ : aA \times aC$. And consequently the Surface of the Sea will go thro' the Points $ikbc$, where bc under the *Æquator* is higher than ik towards the Poles.
Q. E. D.

NB. *That the Centrifugal Force will still add to the Height of the Sea at bc , is plain from what we have said before. And if we apply these Principles to determine the different Lengths of Pendulums, swinging Seconds at Paris and at the Æquator; from the Gravity at Paris, compar'd to the Gravity at the Æquator (in this Supposition of the Action of Gravity and Figure of the Earth) a Pendulum must be shorter at the Æquator by more than 10 Lines, without considering the Centrifugal Force; and if the Centrifugal Force be taken into Consideration, the Pendulums must be shortened near a whole Inch. But this being about five Times more than agrees with Observation; what proves too much, proves nothing at all.*

Having thus shewn, that Monf. *Mairan's* Account of the Action of Gravity, on several Places upon the Earth's Surface, can be of no Service for reconciling the Experiments made on Pendulums, with the Figure of the Earth deduc'd from Monf. *Cassini's* Measures: I proceed to shew that his Demonstrations are founded upon wrong Principles. And first, in Relation to Gravity.

This Gentleman has follow'd Sir *Isaac Newton*, in saying, that Gravity increaseth in a duplicate reciprocal

Proportion of the diminish'd Distance from the Center of the Force, and so *vice versâ*; but he has follow'd Sir *Isaac Newton* no farther than serv'd his present Purpose; otherwise he wou'd have known. — That in respect to a Central Body (as a Planet) towards which others are (*attracted* or) impell'd by Gravity, this Law obtains only, as Bodies attracted, are remov'd from the Surface of the Planet, to greater Distances from the Center compar'd with that Distance; or as from greater Distances they approach nearer to the Planet. — That the greatest Action of Gravity is at the Surface of the Planet. — That afterwards in advancing towards the Center, the Force of Gravity, on the Body attracted, continually grows less, decreasing directly as the Distance; and that this holds true in a Spheroid as well as a Sphere. — That on different Parts of the Surface of the Earth (in the Condition it is now) the Gravity on Bodies is reciprocally as their Distance from the Center of the Earth. — That though at a considerable Distance we look upon the Earth, or any Planet, or even the Sun, as a Point (in the Center of the Forces tending towards it) endued with an absolute Force, proportional to its Quantity of Matter; yet when we come so near the Body as to consider the Space it takes up, we are to take notice, that the whole Attraction or Gravity of the Body, is made up of the Sum of the Attraction of all its Parts properly combin'd; and therefore, that when a Corpuscle, or Body attracted, comes to be within the Planet, or Body attracting, the Matter above it draws it back in such a Manner, that it leaves it only a Force to go on towards the Center, which is directly as the Distance, as we have already said; just as if a Body concentric to the Planet (whether spherical or spheroidical) had its Surface just where the Corpuscle is, and all the exterior Crust or Shell was annihilated.

I do not doubt but Monf. *Mairan* will be of this Opinion, when he has carefully and impartially examin'd the 12th and 13th *Sections* of the First Book of Sir *Isaac Newton's Principia*, and the 18th, 19th, and 20th *Prop.* of the Third Book. And if he will be at the Pains to compare the 38th and 39th *Proposition* of the Third Book with the 66th of the First, he will find that the Precession of the *Æquinoxes* is owing to the broad Spheroidical Figure of the Earth; and that if it had Monf. *Cassini's* Figure, the *Æquinoctial* Points wou'd move *in Consequentia* faster than they do now *in Antecedentia*.

Further, Monf. *Mairan* demonstrates, that in an oblong Spheroid, the Diminution of Gravity, by the Centrifugal Force, encreases faster in going from the Poles to the *Æquator*, than it wou'd do in a Sphere, and faster in a Sphere than it wou'd do in a broad Spheroid; and therefore wou'd shew, " That notwithstanding the Surface of the Earth is nearer to the Center in Monf. *Cassini's* Figure than in Sir *Isaac Newton's*, yet the Centrifugal Force will diminish the Gravity so fast in going from *Paris* to the *Æquator*, that the shortening of Pendulums, to make them swing Seconds at the *Æquator*, may very well be accounted for that Way."

Now let us examine into this Matter, to see whether the Cause is adequate to the Effect.

If the Distance from the Surface of the Earth at the Pole to the Center be 96, and the Distance of the Surface at the *Æquator* be 95, the Distance of the Surface at *Paris*, in the Latitude of $48^{\circ} 50'$, will be 95,562, &c. by the Property of the Ellipse. Now since the Force of Gravity, in different Places on the Earth's Surface, is reciprocally as the Distance from the Center, and the Lengths of Pendulums, that perform their Vibrations in the same Time, are directly as the Force of Gravity; therefore

therefore the Length of Pendulums at *Paris*, will be to their Length at the *Æquator*, as 95 to 95,562, &c. that is, as 440,555, &c. to 443,165, &c. and consequently they must be lengthen'd 2,61 &c. Lines. But as from Monf. *Mairan's* Principles, the Diminution of Gravity by the Centrifugal Force, is greater at the *Æquator* than at *Paris*, hardly $\frac{1}{440}$ Part of the whole Gravity at the *Æquator*, the Pendulums must be shortened in that Proportion; so that then the Length of a Second-Pendulum, will be $440,555 + 2,61 - 1$ Lines. But as that Quantity is greater than 440,555, &c. therefore the Pendulums upon the Whole must be lengthen'd: Nay, though we shou'd allow a shortening of two Lines; since by Observation Pendulums are found to be about two Lines shorter at the *Æquator*, the oblong spheroidical Figure of the Earth cannot be consistent with the Experiments on Pendulums.

I beg Leave to set down Monf. *Mairan's* aforesaid Demonstration here; that we may see whether he has assum'd true Principles.

PROPOSITION V.

“ * XI. *The Centrifugal Force at any Degree of*
 “ *Latitude, taken upon the oblong Spheroid, between*
 “ *the Æquator and the Pole, is less in Comparison*
 “ *to the Centrifugal Force at the Æquator, than it*
 “ *wou'd be at the same Degree of Latitude taken upon*
 “ *a Sphere; or, which is the same thing, the Centri-*
 “ *fugal Force encreases more, going from the Poles*
 “ *towards the Æquator, upon an oblong Spheroid,*
 “ *than upon a perfect Sphere; and consequently Gra-*
 “ *vity diminishes more, and a Pendulum must be more*

* See Monf. *Mairan's Dissertation*, Article XI. &c.

“ *shortened under the Æquator, in the Hypothesis of*
 “ *the oblong Spheroid, than in that of a perfect*
 “ *Sphere.*

“ Having describ’d an oval Curve of any Kind, as
 “ for Example, the Ellipse * ADBE abovementioned,
 “ and inscrib’d the Circle DHE, whose Radius is DC
 “ = half the shorter Axis DE; upon AD take any
 “ Point as R, between the Æquator and the Pole, and
 “ from that Point to the *Evoluta* OTX draw the *Ray*
 “ *of Curvature* RT, which gives the *Line of Tendency*
 “ RP (*Art. IV.*) Draw likewise from the common Cen-
 “ ter C, to the Circumference of the Circle DH, a Ra-
 “ dius CV, parallel to PR, and meeting the Circle at V;
 “ then from the Points R, V, draw the Lines RN, VZ,
 “ perpendicular to the Axis AB.

“ It must be observ’d, *First*, That as the Ellipse AD
 “ represents a Meridian of the oblong Spheroid, the
 “ Circle DH represents a Meridian of a Sphere in the
 “ same Plane.

“ *Secondly*, That the Point V, on the Circular Me-
 “ ridian, answers to the same Degree of Latitude as
 “ the Point R, upon the elliptical Meridian; because
 “ the Lines PR, CV, being parallel to each other, and
 “ perpendicular, the one to the Ellipse and the other
 “ to the Circle (*by Construction*) the touching Planes,
 “ or Horizons of the Points R, V, will also be pa-
 “ rallel.

“ *Thirdly*, Whence it follows that the Diminution of
 “ the Centrifugal Force (acting against Gravity) on ac-
 “ count of its Obliquity to the Horizon (*Art. X.*) of

“ the same Degree of Latitude on the Elliptical and on
 “ the Circular Meridian, is the same in both Cases, and
 “ in the same Ratio as the absolute Centrifugal Forces
 “ represented by the Perpendiculars $R N, V Z$, (*Art. IX.*)
 “ Therefore to know whether the Centrifugal Force
 “ (whether absolute or relative) of the Point R , upon the
 “ oblong Spheroid $A D B E$, be less or greater in respect
 “ to the Centrifugal Force under the common $\text{\AA}$ equator
 “ $D E$, than the Centrifugal Force (whether absolute or
 “ relative) of the correspondent Point V upon the Sphere;
 “ nothing more is requir’d than to see which is the
 “ longest of the two Perpendiculars, namely, $R N$ in
 “ the oblong Spheroid, or $V Z$ in the Sphere; since
 “ these two Lines express the Radii of the Circles of
 “ Revolution, and consequently the absolute Quantity
 “ of the Centrifugal Forces.

“ *4thly and lastly*, That the Ratio of the Centrifugal
 “ Forces of two correspondent Points upon the oblong
 “ Spheroid $A D B E$, and the inscrib’d Sphere $D H E$, to the
 “ Centrifugal Force of their $\text{\AA}$ equators is the same, suppo-
 “ sing the Sphere of any other Bigness; and that it has
 “ been determin’d here of the Diameter $D E$, only to
 “ render the Demonstration easier, by giving the same
 “ Consequent to the Antecedents $R N$ and $V Z$. For
 “ if about the Center C and with the Radius $C d$, the
 “ Circle $d h e$ be describ’d equal (for Example) to a
 “ Meridian of a Sphere of the same Solidity as the ob-
 “ long Spheroid $A D B E$; and the Radius $C V$ be pro-
 “ duc’d till it meet the Circle $d h$ at the Point u , and
 “ $u z$ be let fall perpendicular to the common Axis of
 “ Revolution, and parallel to $V Z$: It is plain, that
 “ we shall always have $V Z : D C :: u z : d C$, or
 “ $\frac{V Z}{D C} = \frac{u z}{d C}$, and consequently $\frac{R N}{D C}$ will have the

“ same

“ same Ratio to $\frac{VZ}{DC}$ as to $\frac{RN}{DC}$.

“ Therefore, in order to demonstrate that the Centrifugal Force of a Point, taken in any Latitude upon the oblong Spheroid, is less when compar'd to the Centrifugal Force of the like Point, taken upon a Sphere in respect to the Centrifugal Force at the Æquator; there is nothing more requir'd than to shew that $RN < VZ$, because by that means we shall have $\frac{RN}{DC}$

“ $< \frac{VZ}{DC}$.

“ This being observ'd; from the Point R, draw the Line RI, parallel to the Axis AB, and meeting the Circle DH at K, and the Diameter DE of the Æquator at the Point I. From the Point K having let fall the Perpendicular KL = RN, upon the Axis AB, and drawn KC to the Center C; the Question will be brought to this, *viz.* To know whether the Point V coincides with the Point K; or whether it is above it towards D, or below towards H.

“ But $CK = CV = CD > PR$ (*Art. VIII.*) therefore CK and PR being both between the Parallels AC, RI, the greatest CK is more inclin'd to them than the least PR, and the Angle KCA is less than the Angle RPA = VCA. And since these two Angles have each of them one of their Sides coinciding with the Line AC, namely, the Side AP of the Angle RPA, and the Side AC of the Angle KCA, it follows that the Side VC of the Angle VCA = RPA $> KCA$, will go above CK between CK and CD, and meet the Line RI at the Point G, between K and J, and the Circle DH at the Point V, which consequently

“ quently will be above RI , between K and D . There-
 “ fore $CV = CG + GV$ is $= PR + GV$, and con-
 “ sequently VZ , which meets RI at the Point F , is
 “ $= ZF + FV = RN + FV$; and therefore RN
 “ $= VZ - FV$. Therefore $RN < VZ$.

“ And because the same Thing may be demonstrated
 “ in respect of any other Point, taken between the $\mathcal{A}$ -
 “ quator and the Pole; and that Gravity, and conse-
 “ quently the Length of a Pendulum diminishes, as the
 “ Centrifugal Force encreases. Therefore *Q. E. D.*

C O R O L L A R Y.

“ XII. From what has been demonstrated, and from
 “ *Prop. 3. Art. VIII.* it follows, that the Perpendicular
 “ which is drawn from any Point of an oval Meridian
 “ to the Axis, will be so much shorter, in Comparison to
 “ the Perpendicular drawn from the correspondent Point
 “ of an inscrib'd circular Meridian, as the Latitude is
 “ greater; and consequently (by *Art. XI. Num. 3.*)
 “ the Centrifugal Force will be so much the less, and
 “ Gravity so much the greater, upon the oblong Sphe-
 “ roid, in respect to the Centrifugal Force, and the Gra-
 “ vity under the $\mathcal{A}$ quator.

“ For as the Line RP does always decrease, as the
 “ Point R is taken nearer to the Pole A , it is evident,
 “ that the Angle VCK will continually encrease, in
 “ respect to the Angles VCA , KCA , as it is their Dif-
 “ ference, and consequently that the Perpendicular VZ
 “ will be so much greater than the Perpendicular
 “ $KL = RN$.

I pass over the Demonstration of the latter Part of his
 Proposition above-mentioned, which he deduces justly
 from his Construction, if what he says (*Num. 2.*) be
 right; because in such a Case it cannot be call'd in Que-
 stion;

tion; and proceed to an Observation that he makes afterwards, viz. “ *We must take care to observe in the foregoing Propositions and Corollaries, that the Comparison is always made between two similar Points of Latitude, taken upon the two Spheroids, or upon one of the Spheroids and the Sphere, between the Æquator and the Poles, in respect to the Centrifugal Force upon the Æquator of any one of these Spheroids, or of the Sphere. For if we only compar’d absolutely the Centrifugal Force of a Point of the Æquator of the one, to the Centrifugal Force of a correspondent Point of the Æquator of the other, it is plain that it would be greater upon a flatted Spheroid than upon a Sphere, or than upon an oblong Spheroid of the same Solidity, in the Ratio of the great Axis of the generating Ellipse of the flatted Spheroid, to the Diameter of the Sphere, or to the shorter Axis of the generating Ellipse of the oblong Spheroid. And in all Likelihood, this must be the Reason that has made others, who have treated of this Subject, to imagine the very contrary of what I have demonstrated.*”

As *Monf. Mairan* considers the Earth at rest, in the Construction for his Demonstration above quoted, and afterwards observes what Effect the Centrifugal Force will have upon Bodies on its Surface, to diminish the Gravity, with which they endeavour to descend in their Line of Tendency RP : He shou’d not only have taken notice (as he has done) that the whole Centrifugal Force NR is not to be subtracted from the Gravity at R , as the whole Centrifugal Force CD is to be subtracted from the whole Gravity at D , because of the Obliquity of RN to PR ; but he should have observ’d also, that the Obliquity of the Plane of the Parallel NR , in which

the Centrifugal Force acts, must alter the Line of Tendency RP , and change the Direction RP into RW , somewhere between the Point P and the Center C ; for if there be a heavy Body as a Plummets, hanging by a Thread in the Line SR , or SP , the Line of Tendency which has been suppos'd perpendicular to the Curve ARD , without taking in the Effect of the Centrifugal Force; as soon as the Spheroid revolves about its Axis, the Body which wou'd fall in the Line SR , acted upon only by one Force, namely, that of Gravity, will now be acted upon by another Force, at the same Time pushing it in the Line Ss (which is the same as Rr) and consequently will move in the Line Sr , diagonal of the Parallelogram $sSRr$; or, which is all one, a Body plac'd at R will have its Line of Tendency in RW , as I have already shewn in my first Dissertation on this Subject; only I did not suppose the Earth a Spheroid before the diurnal Motion, and therefore made use of the Line ZV instead of the Line NR ; so that it may be objected that the Angle rSR will not be so great in a Spheroid as in a Sphere, because the Centrifugal Force which acts with the same Obliquity (since $NRP = ZVC$) is as much less in the Spheroid as NR is less than ZV : But I was aware of that, and therefore made the Angle RSr only of 5 Minutes, when it really appears to be of almost six Minutes, when the Earth is suppos'd spherical; and therefore, without coming to give the exact Quantity of the said Angle, one may easily perceive, that Monsr. *Cassini's* Difference of the Axis and Æquatorial Diameter will produce a Figure, in which the Angle RSr , will not be less than of 5 Minutes.

Such an Obliquity, caus'd in the Direction of Gravity, will render the oblong spheroidical Figure of the Earth impossible, because then Fluids wou'd not have the Lines of their Gravity perpendicular to the Horizons of the

Places where they are, (supposing the Horizons of Places to be Planes touching the Curve of the Earth in those Places) and Plumb Lines wou'd be so far out of the Perpendicular to Lines of Level, as to make an Angle easy to be observed, as I have shewn in my former Paper.

But if the same Cause be suppos'd to act upon the Sea to make it level, as makes heavy Bodies to fall (which certainly must) then indeed Lines of Level will be perpendicular to Plumb Lines, and the Level of the Sea, taken always for the Horizon of a Place, will not be a Plane touching the Earth, but cutting it towards the Poles, and consequently the Water will be carried towards the Æquator, as was before shewn.

Besides, the Difference of the Action of the Centrifugal Force wou'd not be so great between correspondent Points of the same Latitude in the Spheroid and in the Sphere; for when the Line of Tendency RP is by the Centrifugal Force chang'd into RW , the Point R upon the Spheroid does no longer correspond in Latitude with the Point V upon the Sphere, but must be taken nearer to V , so that the Line RW may become parallel to VC , and $RWA = VCA$.

If it be alledged here, that Monf. *Mairan* supposes the Earth in Motion, and takes in the Effect of the Centrifugal Force, when he makes the Line of Tendency to be RP ; I answer, that if he had consider'd the Earth as revolving upon its Axis, he wou'd not have made VC the Line of Tendency of a spherical Earth in Motion, since it is the Line of Tendency of such an Earth at rest.

In Monf. *Mairan*'s Observation above-mentioned, he says, " that we are not to compare the Centrifugal Force at the Æquator of an oblong Spheroid, with the Centrifugal Force at the Æquator of a Sphere, or at the Æquator

“Æquator of a flattened Spheroid of the same Solidity ;
 “allowing that then it wou’d be greater in the Sphere,
 “and still greater in the flattened Spheroid : but only the
 “Centrifugal Forces in several Latitudes upon the same
 “Figure.” —But I beg Leave to differ from him for the
 following Reasons.

First, Because the Force of Gravity is not the same at the Æquator of the flattened Spheroid, as it is at the Æquator of the Sphere, or as it is at the Æquator of the oblong Spheroid.

Secondly, Because it is not the same in different Latitudes, in either of the Spheroids. (See Sir *Isaac Newton* Lib. 3. Prop. 19 and 20.) And *Monf. Mairan’s* Way of arguing will only serve, in Case the Gravity shou’d be the same in all the Points of the Surface of the Earth in his Figure, and also in the two other Figures.

For Example, let the uniform Gravity be call’d g ; and *First*, let the Centrifugal Force at the Æquator of the flattened Spheroid be call’d $c + 2$; and the Centrifugal Force in any Latitude, as for Example, the Latitude of *Paris* (as it is diminished on Account of a shorter Co-sine of Latitude, and likewise on Account of its Obliquity to the Line of Tendency,) be call’d $c + 2 - l$; the Difference of the Diminution of Gravity at *Paris*, and at the Æquator will be $\overline{g - c + 2} - \overline{g - c + 2 - l} = l$.

Secondly, Let the Centrifugal Force at the Æquator of the Sphere be call’d $c + 1$, and the Centrifugal Force at the Latitude of *Paris* be call’d $c + 1 - l + m$; the Difference of the Diminution of Gravity at

T t 2

Paris,

Paris and at the *Æquator* in a spherical Earth, will be

$$\overline{g - c + 1} - \overline{g - c + 1 - l + m} = l + m.$$

Thirdly, Let the Centrifugal Force at the *Æquator* of the oblong Spheroid be call'd c , and the Centrifugal Force at *Paris* be call'd $c - \overline{l + m + n}$; the Difference of the Diminution of Gravity at *Paris*, and at the *Æquator*, in an oblong spheroidical Earth, will be $g - c -$

$$\overline{g - c - l + m + n} = l + m + n.$$

Now, if Gravity shou'd in every Case be equal to g , it is evident, that the shortening of Pendulums, at the *Æquator*, wou'd be greater in the oblong Spheroid, than in the Sphere, or in the flatted Spheroid; because as the Lengths of Pendulums diminish with the Gravity, those Lengths will be at *Paris* and at the *Æquator*, when compar'd, as $g - \overline{c + 2 - l}$ to $g - \overline{c + 2}$ in the flatted Spheroid; as $g - \overline{c + 1 - l + m}$ to $g - \overline{c + 1}$ in the Sphere, and as $g - \overline{c - l + m + n}$ to $g - c$ in the oblong Spheroid; and consequently, from what *M. Mai-
ran* has demonstrated this Ratio of $g - \overline{c - l + m + n}$ to $g - c$, being greater than either of the others, the Pendulums must be shortened in the oblong Spheroid.

But as the Force of Gravity is less at the *Æquator* of the flatted Spheroid, than at the *Æquator* of the Sphere, or of the oblong Spheroid of the same Solidity: let us express its Quantity in the three Cases by $g - s$, g , and $g + s$, and we shall then find the Lengths of the Pendulums, at the *Æquator* of the three Solids, as $g - s - \overline{c + 2}$, $g - \overline{c + 1}$, and $g + s - c$; consequently the Lengths of
Pendulums

Pendulums will be greatest at the Æquator of the oblong Spheroid, because $g + s - c$ is the greatest Quantity.

Lastly, To compare the Lengths of Pendulums at the Æquator of the oblong Spheroid, thus found, with their Lengths at the Latitude of *Paris* upon the said Spheroid — Let us express the Excess of Gravity at the Æquator, whereby it is greater than at *Paris* (because in this Figure, *Paris* is farther from the Center of the Earth, than the Æquator, by $\frac{1}{10}$ Part) by the Letter s , and the Excess of the Centrifugal Force at the Æquator, above that Part of it which acts directly against Gravity at *Paris*, by $l + m + n$, the Gravity at *Paris* by g , and the Centrifugal Force at the Æquator by c ; then $g + s - c$ will still represent the diminish'd Gravity, and answer to the Length of Pendulums at the Æquator,

whilst $g - \overbrace{c - l + m + n}$ or $g - c + l + m + n$ represents the diminish'd Gravity, and consequently the Length of Pendulums at *Paris*. If s be equal to $l + m + n$, Pendulums will be as long at the Æquator as at *Paris*; and if s be greater than $l + m + n$, Pendulums will be longer at the Æquator. But making all possible Allowance, in Favour of Mons. *Mairan*'s Hypothesis, no Calculation will bring $l + m + n$ to be greater than, or ever equal to s . Therefore Mons. *Mairan*'s Demonstrations, above-mentioned, are of no Force to prove the Earth to be an oblong Spheroid.

And now, I think, I have answer'd all that relates to the Figure of the Earth in Mons. Mairan's Dissertation; in shewing, That his Conjectures can neither be supported by those Physical Principles which Sir Isaac Newton has Mathematically deduc'd from unquestion'd

questioned Observations and Experiments accurately made; nor even by those Principles which He (M. Mairan) has assum'd to serve his intended Purpose — That his Demonstrations, relating to the Difference of the Action of the Centrifugal Force, are of no Service to him, for reconciling the Experiments made on Pendulums, with Mons. Cassini's Measures; — because, when applied to Sir Isaac Newton's Principles, they will make Pendulums longer at the Æquator than at Paris, and when applied to Mons. Mairan's own Principles, they will make them a whole Inch shorter at the Æquator than at Paris, contrary to all Observations, which, at a Medium, make Pendulums but about two Lines or $\frac{1.66}{10000}$ of an Inch longer at the Æquator than at Paris. — That he has built his Demonstrations upon a wrong Notion of Gravity — And that he has not consider'd what is most material in the Effect of the Centrifugal Force, acting on Bodies descending by their Gravity, between the Æquator and the Poles, namely, the Alteration of their Line of Direction, which wou'd make them fall out of the Perpendicular towards the Æquator.

I shall add one more Philosophical Argument, given me by a Friend, to whom I communicated my Thoughts on this Subject; because it is wholly independent on those Principles of Philosophy, concerning which, some of the Gentlemen that believe the oblong spheroidical Figure of the Earth, and the *English* Philosophers, are not yet agreed; and it is this.

If the Earth was of an oblong spheroidical Figure, higher at the Poles than the Æquator; the Axis of its Revolution, wou'd either go thro' one of its short Diameters, or be continually changing, unless the said Axis did exactly coincide with the Axis of the Figure.

DEMONSTRATION.

Suppose such an oblong Figure as Aa^* fix'd to the Axis Pp at the Center C , but capable of moving freely round it towards P or towards p , yet so as to be oblig'd to move with the Axis, when it is turned round. Suppose now the Poles P and p to be fix'd, and the Body, thus constituted, to be turn'd swiftly round the Axis Pp ; then if the Angle ACP be oblique, and the Figure $ADaE$ be oblong, the Parts AC and Ca will acquire a Centrifugal Force, which will enlarge the Angle pCA , till it comes to be a right one. Besides this, a Velocity will be generated in the Motion, while A is going towards the Perpendicular aC , which will make it go farther on towards P , as to B , with a Motion which will after that be retarded, till the Centrifugal Force has Strength enough to send it back again the contrary Way; and so it will move continually with a reciprocal Motion, like the Oscillation of a Pendulum; and if a little of this Motion be lost at every Oscillation, then the oblong Figure $ADaE$ will at last move quietly about its lesser Axis DE coinciding with Pp .

If Aa did not at first exactly coincide with Pp , the Centrifugal Force will have the above-mentioned Effect; and that this is not the Case in the Earth is more than probable, because the unequal Distribution of Sea and Land, besides the Phænomena of the Tides must make the Axis of its Gravity, and consequently the Axis of its Revolution, to differ from the Axis of the oblong Spheroid, if the Earth had such a Figure; without considering that every Earth-quake wou'd alter so nice an Equilibrium, which once lost, wou'd never be recover'd again.

To leave nothing unexamined, relating to the Controversy, I have again consider'd the Measures and Observations, mentioned in the Account of the Meridian drawn thro' *France*, in the Memoirs of the *Royal Academy*, for the Year 1720; and I find them to want a great deal of the Accuracy requir'd in so nice a Point, as determining the different Lengths of Degrees upon the Surface of the Earth. To prove my Assertion, I beg that the Reader will examine the following Tables, whereby it appears, that if any thing certain can be deduc'd from the said Observations and Measures, (either taken as they are, or reduc'd to the Level of the Sea, by the Rules given by *Monf. Cassini**) it will be in Favour of Sir *Isaac Newton's* Figure of the Earth, rather than theirs.

In the following Table, the first Column gives the Names of Places; the second, the Distances from *Paris*, according to the Measures taken by the *French* Gentlemen; the third, the Latitudes observ'd by the same; the fourth, the Latitudes, such as the measur'd Distances will give them, supposing the Earth spherical; the fifth, the Differences between these and the Latitudes observ'd, express'd in Seconds of a Degree, where when the Latitude computed, exceeds the Latitude observ'd, the Letter N (North) shews that Difference to be in Favour of *Monf. Cassini's* Figure, and the contrary Difference mark'd by the Letter S (South) is in Favour of Sir *Isaac Newton's* Figure.

* *Memoirs for the Year 1720. Vol. I. P. 1. Ch. 13.*

Names of Places.	Distances from <i>Paris</i> measur'd.	Latitudes observ'd.	Latitudes in a spherical Earth computed from the measur'd Distances.	Differences in Seconds.
I.	II. Toises.	III.	IV.	V.
Dunkirk.	125552	51° 21' 25 $\frac{1}{2}$ "	51° 21' 25 $\frac{1}{2}$ "	0''
Amiens.	60370	49 53 48	49 53 48	0''
Sourdon.	4997 $\frac{1}{2}$	49 42 42	49 42 52,1	10,1 N
Paris.		48 50 10	48 50 20,3	10,3 N
Malvoisine.	18838	48 30 47	48 30 32,1	14,9 S
Voufon.	67962	47 39 17	47 38 53,6	23,4 S
Bourges.	100192	47 4 31	47 04 58,7	27,7 N
S. Sauvier.	139934	46 23 24	46 23 12	12,0 S
Croc.	169540	45 51 43	45 52 4,6	21,6 N
Bort.	196484	45 23 27	45 23 45,2	18,2 N
Aurillac.	223606	44 55 13	44 55 14,5	1,5 N
Rodès.	256575	44 20 54	44 20 35,1	19,9 S
Alby.	280612	43 55 32	43 55 19	13,0 S
Carcaffone.	321430	43 12 55	43 12 24,5	31,5 S
Collioure.	360604	42 31 13 $\frac{2}{3}$	42 31 13,8	0.

In this Table it is to be observ'd that there is an equal Number of Differences mark'd N (North) and S (South) and if the Differences on each Side be added together, there will be 89'' $\frac{1}{4}$ on the North Side, and 114'' $\frac{7}{8}$ on the South : This last agrees best with Sir *Isaac Newton*'s Figure, which must be supposed for the Correction of so great a Difference.

In the next Table, the first Column gives the Names of Places; the second, the Latitudes observ'd; the third, the Distances in the Meridian from *Paris*, reduc'd to the Level of the Sea; the fourth, the Differences of the second Column express'd in Seconds of a Degree; the fifth, the Differences of the Numbers in the third Column; and the sixth, the Measure of a Degree by the fourth and fifth Columns compar'd.

I. Names of Places.	II.			III. Toifes.	IV. Seconds of a De- gree.	V. Toifes.	VI. Toifes.
Dunkirk.	51°	2'	19"	125454	4103'	65010	57040
Amiens.	49	53	56	60444	1859	29416	56965
Clermont.	49	22	57	31028	1967	31028	56787
The R. Observatory.	48	50	10	0	4253	67959	57525
Voufon.	47	39	17	67959	4553	71978	56912
St. S. Sauvier.	46	23	24	139937	1901	29602	56058
Croc.	45	51	43	169539	1677	26941	57834
Bort.	45	23	46	196480	1713	27136	57028
Aurillac.	44	55	13	223616	2060	32858	57422
Rodés.	44	20	53	256474	1521	24138	57131
Alby.	43	55	32	280612	2557	40818	57468
Carcaffone.	43	12	55	321430	2502	39184	56380
Collioure.	42	31	13	360614			

In this Table in the third Column, over-against *St. Sauvier*, the Number which was 139944 is corrected to make it 139937, to the Advantage of the oblong Figure. In the sixth Column, the Numbers appear so irregular, as to be unfit to decide this Controversy: Then if a Comparison be made between *Dunkirk*, *St. Sauvier* (which is very near the Middle of *France*, and almost in the Meridian of *Paris*) and *Collioure*, the Measurement is absolutely in Favour of Sir *Isaac Newton's* Theory; the mean Degree between *Dunkirk* and *St. Sauvier* being larger by about 64 Toifes, than between *S. Sauvier* and *Collioure*; and to reduce them even to an Equality, there must be a greater Alteration made in the Situation of those three Places, than it is reasonable to suppose their Observations to be capable of admitting. Here follows the Comparison.

<i>Dunkirk</i> and <i>Collioure</i>	A mean Degree is	57061
<i>Dunkirk</i> and <i>Paris</i>		56960
<i>Paris</i> and <i>Collioure</i>		57097
<i>Dunkirk</i> and <i>S. Sauvier</i>		57090,4
<i>S. Sauvier</i> and <i>Collioure</i>		57026,5
According to Monf. <i>Picard</i> ,		57060

To conclude, I will propose a Method of observing the Figure of the Shadow of the Earth in Lunar Eclipses, whereby the Difference between the Diameters in the oblong spheroidical Figure, if there be such an one as Mons. *Cassini* affirms (*viz.* of 96 to 95) may be discover'd.

Let $P \text{ } \mathcal{A} P \text{ } \mathcal{A}^*$ represent the Earth, seen from the Sun at the Time of the Summer Solstice; it is evident, that the same Figure will express the Section of the Earth's Shadow at the Moon's Distance, as seen from the Earth. If EE represents the Ecliptick, $\mathcal{A} \mathcal{A}$ will be the shortest Diameter of the Section; and if LL be taken for the Moon's Way, in a total and central Eclipse of the Moon, by observing the Time which is spent in the Passage of the Center of the Moon, thro' the Shadow, and reducing that Time to Seconds of a Degree of a great Circle of the Heavens, we shall have the least Diameter of the Shadow.

Again, let the same Letters† represent the same Things, only here the Section of the Shadow is such, as the Earth will cast at the $\mathcal{A}$ equinox, and the Eclipse of the Moon is here supposed partial, its Center just touching the Shadow. When the Moon's Center is got to c , if the Latitude of its Center or its Distance from the Ecliptick be observ'd, we shall have the Length $c C$ nearly equal to the longest Semi-diameter of the Shadow.

Now, comparing $c C^*$ in this Figure to $L C^*$ in the former (the Difference between $c C$ and CP (*Fig. 7.*) and between CL and $C\mathcal{A}$ (*Fig. 6.*) not being worth notice) they ought to be to one another, as 96 to 95, which in such a Shadow will give a Difference of about 25'' at a Medium, sensible enough to be observed, notwithstanding the *Penumbra*. If therefore those A-

* Fig. VI.

† Fig. VII.

* Fig. VII.

* Fig. VI.

astronomers who have Instruments nice enough, and sufficient Skill in the Management of them, to take Angles to 3 or 4 Seconds of a Degree, will observe what I have been mentioning in total and partial Eclipses of the Moon; by such Observations they will easily convince us, that the Figure of the Earth is such as *Monf. Cassini* supposes it, or convince him that he has been mistaken.

*The Semi-diameter of the Earth's Shadow, when the Earth is in Perihelio, and the Moon in Apogæo is 38', or 2280'', without considering the Encrease of the Shadow, on account of the Atmosphere of the Earth, which wou'd make it 39' or 2340'' (allowing one Second for a Mile;) and the Semi-diameter of the Shadow, when the Earth is in Aphelio, and the Moon in Perigæo is 46', 20'', or 2780'', which encreased on account of the Atmosphere of the Earth, will bring it to 47', 20'' or 2840''. Now if the Proportion of 95 to 96 be taken in both Cases you will have these Analogies, $\left\{ \begin{array}{l} 95 : 96 :: 2340'' : 2364'',6 \\ 95 : 96 :: 2840'' : 2869'',8 \end{array} \right\}$ So that $2364'',6 - 2340'' = 24'',6$ will be the Difference of the Semi-diameters, when the Section of the Shadow is the least, and $2869'',8 - 2840'' = 29'',8$ will be the Difference of Semi-diameters, when the Section of the Shadow is the greatest; the Sum of those Differences $24'',6 + 29'',8$ halved, will give the Difference, when the Section of the Shadow is at a Medium $= 27'',4$; from which if we take $2'',4$ because in Fig. 7. C c is a little less than CP, and in Fig. 6. LC is something greater than AC, we shall have C c in Fig. 7. to compare with LC in Fig. 6. which will exceed it by $25''$, if *Monf. Cassini's* Figure of the Earth be the true one.*

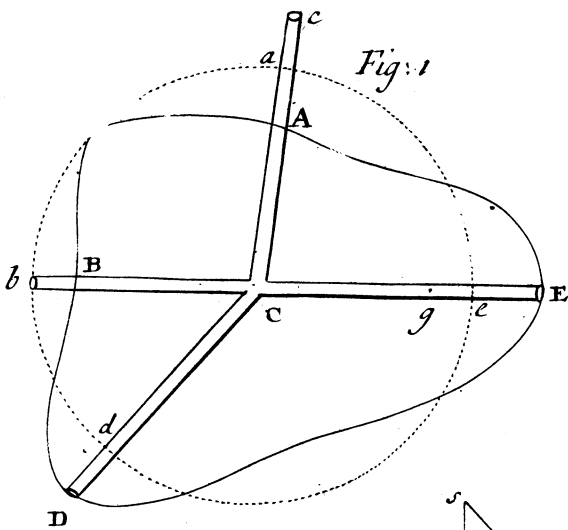


Fig. 1

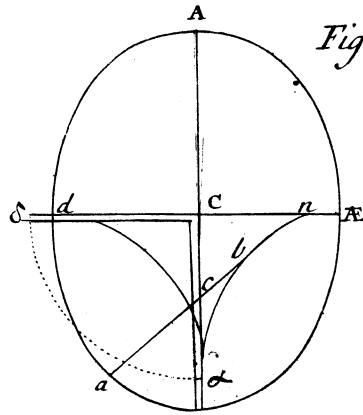


Fig. 2.

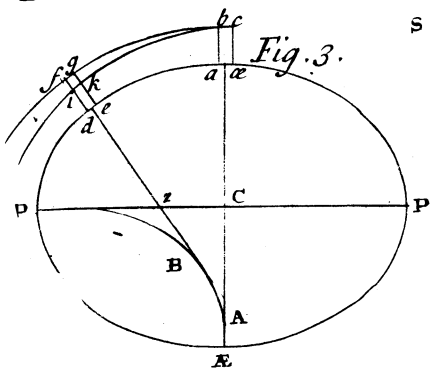


Fig. 3.

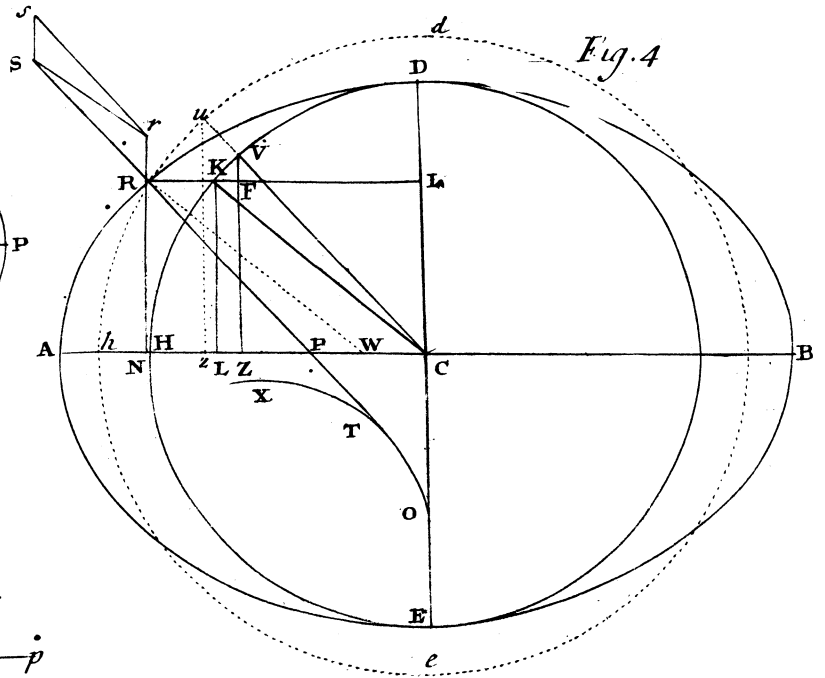


Fig. 4

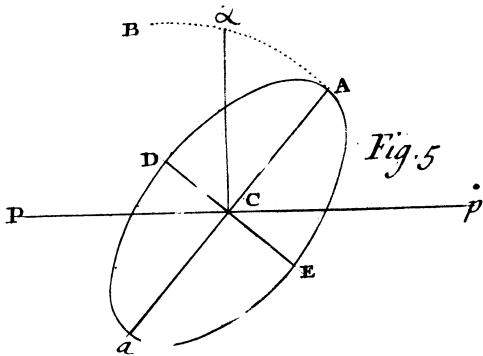


Fig. 5

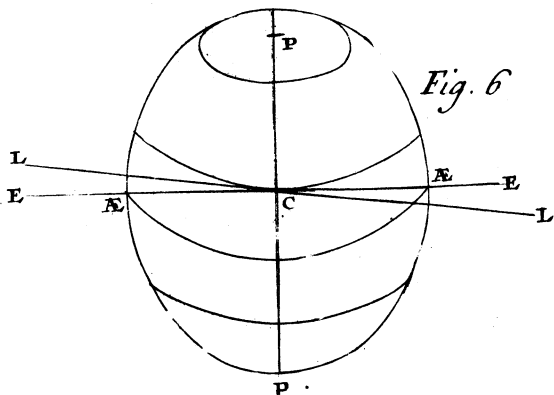


Fig. 6

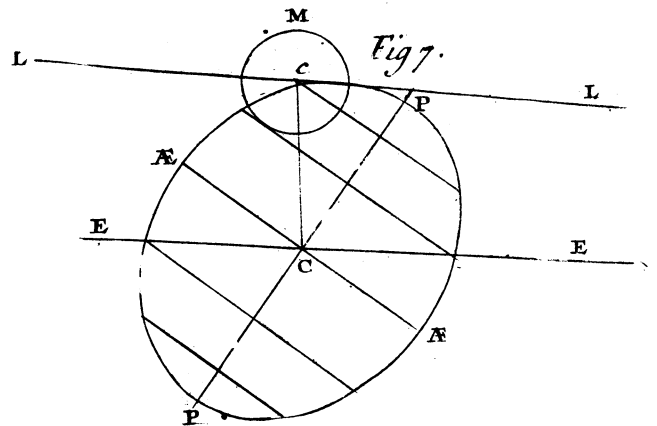


Fig. 7.